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A Review on Numerical Methods for Food Freezing Time Estimation

Mahendra Kumar Sharma[1]; Mohan Kumar Sinha[2*]; Sanoj Kumar[3]
[1]Asstt. Prof. Cum Jr. Scientist, B.A.C., Sabour, B.A.U., Sabour, Bihar
[2*]Asstt. Prof. Cum Jr. Scientist, C.O.H., Noorsarai, B.A.U., Sabour, Bihar
[3]Head, Deptt, of Agril.Engg., B.A.C., Sabour, B.A.U., Sabour, Bihar
[*]Corresponding Author (Mohan Kumar Sinha) Email: mohansinhabau@gmail.com

***Abstract:** Estimation of the freezing times for different types of foods with different shapes is a complex problem in reality. However, simplifications based on practical assumptions usually lead to estimations with an acceptable accuracy. In this regard, some analytical solutions are available for simple geometric shapes while numerical solutions were given for more complicated shapes.This article presents various numerical methods that estimate the freezing time for different products. In this regard, the freezing process is mathematically modelled by transient heat conduction equations that incorporate the physical properties of the three distinct regions that exist during a freezing process. These regions are namely, the solid phase region, the liquid phase region and the interface region. Methods for calculating freezing time and the evolution of temperature and phase change are reviewed and their underlying assumptions and limitations are critically examined.*

***Keywords:** Food freezing, Transient heat conduction, Finite difference, Finite element, Finite volume.*

Introduction

Food preservation processes (e.g. food refrigeration) appear to be energy intensive applications. Food freezing effectively reduces the activity of micro-organisms and enzymes, thus retarding deterioration. Rapid freezing or thawing of foods that mainly contain water can be achieved by high pressure freezing or thawing processes. This is due to the fact that the melting temperature of water decreases with pressure. Moreover, significant preservation of the microstructure biological substances is permitted via pressure shift freezing. High pressure freezing and thawing applications in foods are reviewed by LeBail et al. (2002). In addition, crystallization of water reduces the amount of liquid water in food items and inhibits microbial growth. In order for freezing operations to be cost-effective, refrigeration equipment should fit the specific requirements of a particular freezing application. In this regard, Cleland and .Ozilgen (1998) reviewed the thermal design calculations for food freezing equipment to answer the questions that would be most needed by who design, build and commission freezers. The design of such refrigeration equipment requires estimation of the freezing times of foods, as well as the corresponding refrigeration loads. An accurate method for predicting temperature history during melting or solidification would be useful tool for energy saving and consequently a cost reducing tool.

Estimation of the freezing times for different types of foods with different shapes is a complex problem in reality. However, simplifications based on practical assumptions usually lead to estimations with an acceptable accuracy. In this regard, some analytical solutions are available for simple geometric shapes while numerical solutions were given for more complicated shapes. Many of these solutions are available in Arpaci (1966), Plank (1943), Cleland and Earle (1979, 1984), Hung and Thompson (1983), Hossain et al. (1992a–c), Carslaw and Jaeger (1959), Wilson and Singh (1987), Kim and Kaviany (1990), ASHRAE (1993), and Dincer (1997).

A significant number of problems that quantitatively describe physical phenomena can be generally reduced to a system of ordinary or partial differential equations with appropriate initial and boundary conditions, valid for a certain region or domain. Numerical analysis can adequately describe the problem under consideration and achieve results of high accuracy.

Numerical methods are regularly used to model heat transfer during food freezing processes. The advantage of numerical methods over simple equations is that effects of the phase change over a range of temperature, changing thermal properties and heterogeneity of food products can be analyzed. If numerical methods are formulated and implemented correctly to reduce numerical truncation and rounding errors, they are generally considered to be the most accurate, reliable and versatile freezing and thawing time prediction methods (Cleland et al., 1987).

Simple analytical formulas developed for estimating the freezing time can predict the time with minimal errors but often we need to know much more about what happens during processing - temperature, moisture, water activity to predict the quality of the product. For this we have to use numerical methods, which have become popular in the food industry in the last two or three decades with the wide availability of computers.

Finite Difference Method (FDM)

FDM is the easiest and fastest numerical method. The product is represented by a (usually) regular orthogonal grid of nodes connected by heat conductors, similar to an electrical grids of resistors and capacitors. The equations of heat conduction become discretized and become similar to those describing an electrical network of capacitors and resistors. Each capacitor represents the heat capacity of a subvolume of product, while each resistor represents the heat conduction path between the centres of these subvolumes. This gives a system of equations which can be written in matrix form (1).

$$C\frac{dT}{dt}+KT=f \tag{1}$$

where T is a vector of nodal temperatures, C is the global capacitance matrix containing the specific heat c, K the global conductance matrix containing the thermal conductivity k, and f the global forcing matrix containing known terms arising from boundary conditions. This equation is solved for each small time step until the process is finished. FDM is the earliest form of numerical method to be used. Its advantages are (1) It is easy to understand and to program, and (2) It is very fast, especially in one and two dimensions.

Its main disadvantage is that it can be used only for regular geometries (slabs, cylinders, spheres, brick shapes etc.)

For example, FDM can be easily applied to the modelling of cartoned products. Other products can also be modelled by a similar regular shape. For example, a steakslice of any shape can be modelled by an (infinite) plate, since one dimension is much smaller than the other so heat transfer is practically in 1-D. A leg of lamb or beef can be modelled by a cylinder or perhaps a sphere. A whole beef side has been modelled as a combination of plates and cylinders(Davey et. al., 1997). If the approximate shape is reasonable then the results can be quite accurate.

In numerical methods, heat diffusion equation can be expressed in the following two ways (Pham, 1985; Lind, 1991):

$$c(T)\frac{\partial T}{\partial t} = \frac{d}{dt}\left[k(T)\frac{dT}{dx}\right] \tag{2}$$

and

$$\frac{\partial H}{\partial t} = \frac{d}{dt}\left[k(H)\frac{dT(H)}{dt}\right] \tag{3}$$

Equation (2) is based on methods with temperature as the only dependent variable, while Equation (3) represents the enthalpy methods, which have two dependent variables with enthalpy being the primary and temperature the secondary variables. In Equation (2), the latent heat is represented by a small but finite wide peak of the curve c(T). If time increments are large, the temperature at the node may pass over the range of temperatures at which freezing occurs in only one step, then the latent heat is ignored, and the total time obtained is smaller than the actual one. For avoiding this problem very small time increments must be used. The enthalpy method requires either explicit technique with the consequence of problems of convergence, or implicit procedures in which iteration at each time step is used and is less efficient in terms of computation time. Pham (1985) proposed a method that combines the positive characteristics of both enthalpy and temperature methods. Mannapperuma and Singh (1988) used an explicit numerical method, involving enthalpy formulation to predict temperature distribution in foods during freezing and thawing. The method showed good agreement of calculated freezing and thawing times with experimental values for slab-shaped, cylindrical and spherical products. With proper selection of time increments, the method provides a fast computational procedure for predicting freezing times. Mannapperuma and Singh (1989) extended the enthalpy formulation approach to two- and three-dimensional geometries for fixed temperature, fixed heat flux and convective boundary conditions, and the computer program developed could evaluate the freezing and thawing times and relevant properties of food satisfactorily.

Among the numerical methods, there are finite difference (FDM), finite elements (FEM), and boundary elements and finite control volumes techniques. The first two techniques are most frequently used. The potential contribution of the boundary element technique is significant but only applied to a limited number of food and agricultural engineering problems (Puri & Anantheswaran, 1993).

Finite difference method includes: (i) simple explicit schemes where thermal conductivity (k) and heat capacity (cp) are combined and thermal diffusivity is taken as a function of temperature; (ii) explicit solutions where k and c are taken as separate functions of temperature; (iii) explicit difference formulae based on the enthalpy transformation; (iv) fully implicit, two time level implicit schemes and three time level implicit solutions (Cleland, 1985). Bonacini and Comini (1971) and Cleland and Earle (1984) considered various finite difference schemes for phase change problems and concluded that the implicit, three time level Lees scheme (Lees, 1966) was most accurate. This scheme is conditionally stable and convergent. Formulations of both FDM and FEM based on the Lees scheme have been developed and implemented to predict freezing and thawing for a range of geometries (Cleland et al., 1987). For including the volumetric expansion during freezing, Sheen and Hayakawa (1990) developed a new finite difference model for irregular domains by applying the alternate direction implicit (ADI) central difference and finite volume methods. The model could be used for simulating different thermal processes of heat conduction in foods.

Finite Element Method (FEM)

FEM is probably the most popular method for modelling heat transfer and various other physical phenomena. It consists of dividing the product into subvolumes or elements, each of which contain some nodes which represent points in the solid. As in FDM, equations are set up to describe the heat flow between the nodes. FEM equations are more time consuming to set up than FDM equations and take a longer time to solve. However, FEM can easily handle complex shapes and composite products (for example, meat with bone, fat and lean meat, or a carton with cardboard, air gaps and food). The discretization of the product into elements can be automated, so all the user has to do is to enter the product's shape (using some graphical interface) then tell the computer program to mesh.

Some researchers write their own FEM programs to get maximum flexibility and speed of execution. For example, FEM was used to model a beef side with a series of “slices” (Davey and Pham, 2000). Most people will use one of the several commercial FEM packages available, such as COMSOL (formerly FEMLAB), ANSYS, ABAQUS and NASTRAN.

The finite element method has been used by various authors (Bonacina, Comini, Fasano, & Primicero, 1973; Comini, del Giudice, Lewis, &Zienkiewicz, 1974; Rebellato, del Giudice, & Comini, 1978; Purwadaria & Heldman, 1982; Abdalla& Singh, 1985). For unidirectional and regular geometry problems, FEM does not offer more advantages than the finite difference technique (Ramaswamy & Tung, 1984; Chau & Gaffney, 1990). Moreover, by applying the numerical grid generation approach, finite difference method can be used for irregular geometry as effectively as the more complicated finite element method without sacrificing its simplicity (Ansari, 1999). Both FDM and FEM are widely used as tools to develop simple predictive models.

An alternative approach to the finite element method is the boundary-fitted grid (BFG) method used by Califano and Zaritzky (1997). The technique was applied to simulate the freezing process in two-dimensional systems of arbitrary shape. The results indicated that not only can similar accuracy to that of finite element formulation be obtained, but also very short computer times and small memory requirements are needed.

The Finite Volume Method (FVM)

In FVM grid, the product is again divided into volume elements (as in FEM), and the thermal capacity of each volume element is assumed to be concentrated at its centre, or node. Each node is connected to surrounding nodes by heat conduction links, just as in FDM, except that the grid does not have to be regular. FVM is therefore just as flexible as FEM with respect to product shape.

Computational Fluid Dynamics (CFD) Models

CFD models calculate the fluid flow and temperature around the products as well as inside it. They may discretize space using FDM, FEM or FVM, although the latter two are most often used. In non-solid regions, the equations of fluid flow must be solved to calculate fluid velocities. The great advantage of CFD is that they allow the heat transfer coefficients to be calculated rather than guessed or measured experimentally. So, in principle, the rate of cooling and freezing for any product in any situation can be predicted without doing any experiment, provided the product's properties are known.

The biggest problem with CFD programs is that the flow is usually turbulent, i.e. subject to random and very fast fluctuations. These random fluctuations cannot be solved from first principles. They are very complicated and depend highly on the geometry of the flow. They must be solved approximately by a so-called turbulence model. For example, in the k-ε model, transport equations for the turbulent kinetic energy k and the turbulent dissipation rate ε are set up and solved. To a large extent these models are not rigorous, and they require empirical parameters obtained from experiments. Therefore the results cannot be guaranteed to be accurate. This is particularly so when the flow is highly swirling, or when there is a large amount of recirculation.

The second problem with CFD is that it is very timeconsuming to run. Due to the nature of the fluid flow equations, a very fine grid has to be used, usually millions of nodes or elements. The number of equations to be solved at each time steps is even greater, and they have to be solved by an iterative method. For example, to solve forthe chilling of a beef side above (20 hours in real time)takes about a week on a supercomputer. Due to these problems, a full CFD solution is usually not the best method to use at the moment. Instead CFD can be used to calculate the surface heat transfer coefficient, which is then used as input parameter for a FDM or FEM program that calculate heat conduction inside the product only.

Calculation of heat balance as part of the numerical method implementation is one of the best ways to check the accuracy of all the numerical approximations. The calculated heat flow through the surface of the object should equal the change in the internal enthalpy of the object. Agreement between predicted and experimental values within 2% should be achieved for a numerical method to be considered as accurate and valid (Cleland et al., 1987).

Imprecise predictions of experimental freezing times by correctly formulated finite difference or finite element schemes are largely due to uncertainties in thermal data or imprecise knowledge of the experimental conditions, particularly the surface heat transfer coefficient. If reliable thermal data are used, major contributor of the inaccuracy will be the experimental conditions. In these circumstances numerical calculations provide a reasonable estimate of experimental uncertainty. However, verification of thermal data sets is still a necessity (Cleland, 1985). More recently, Saad and Scott (1997) analysed the accuracy of numerical simulation of one-dimensional freezing problem by providing a technique distinguishing numerical errors from analytical biases. The proposed analytical technique is useful since it can be applied to various types of non-linear heat transfer problems.

An alternative to programming would be to use a commercial heat transfer package. Wang and Kolbe (1994) analysed food freezing in a plate freezer by using a PC-based finite element package. The prediction agreed reasonably well with measured data, which validates the applicability of computer program simulating the freezing process. Solutions of this kind have great potential for food process modelling.

Conclusion

A review of numerical food freezing time estimation methods for food items was given in this paper. Freezing continues to be the most popular and effective method for the long-term preservation of high quality food, and is likely to remain so in the future. Food freezing is a complicated process, involving heat transfer, ice nucleation, ice growth, ice distribution, and the changes in food physical and chemical properties.Although commercial packages are being constantly developed, simplified and rapid predictive methods are still preferred by the industry. This may be attributed to the difficulties in finding appropriate product properties and computational conditions in order to use this kind of software. In this sense, development of simple and accessible software would be very helpful.

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